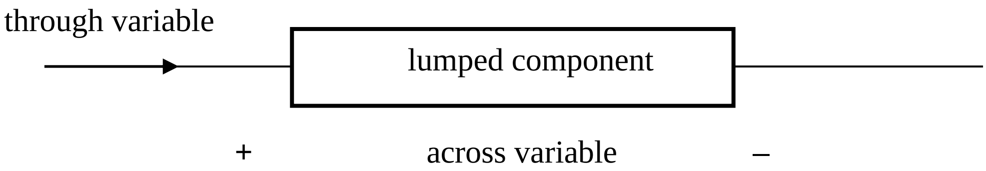
Appendix A: Electroacoustic Modelling

Where feasible, engineering systems are analysed as lumped-parameter models, or as distributed-parameter models if the signal propagation time is significant. The components of a lumped-parameter model have two or more terminals and are joined at nodes to form a lumped circuit or network. The physical state of a two-terminal component is described by a pair of variables (both functions of time t): the through variable (such as electric current $i(t)$) and the across variable (such as voltage difference $v(t)$). Their reference directions are associated in such a way that the product of the through and across variables for any two-terminal component is equal to the instantaneous power (symbol p, unit watt [W]) delivered to the component:

$$p(t) \;=\; i(t)v(t) \quad \text{[W]}.$$

Resistive components dissipate the power as heat whereas capacitive and inductive components store the power as energy or work (symbol w, unit joule [J]) of various kinds.



Sources of power (active components) are included in the above description; the only difference is that the instantaneous power may be negative, that is, the functional relationship between v and i may lie in the second or fourth quadrants of the "volt-amp" plane.

In a complete circuit the through variables obey Kirchhoff's current law (KCL) and the across variables obey Kirchhoff's voltage law (KVL) regardless of the type of components that make up the circuit.

The through and across variables for the three analogous physical systems relevant to electroacoustic modelling are given in the table below:

System	Through Variable	Across Variable
Electrical	current i [A]	voltage difference v [V]
Mechanical (translational)	force f [N]	velocity difference u [m/s]
Acoustical (fluid-pneumatic)	volume velocity U [m^3/s]	pressure difference p [Pa]

In the electrical domain the three basic two-terminal lumped components are the resistor, the capacitor and the inductor. Their constitutive (volt-amp) relationships are:

$v(t) \;=\; Ri(t)$ where R (unit ohm [Ω]) is the resistance of the resistor.

$v(t) \;=\; \dfrac{1}{C}\displaystyle\int_{-\infty}^{t} i(t')\,dt'$ or $i(t) \;=\; C\dfrac{dv}{dt}(t)$ where C (unit farad [F]) is the capacitance of the capacitor.

$v(t) \;=\; L\dfrac{di}{dt}(t)$ where L (unit henry [H]) is the inductance of the inductor.

There are lumped components analogous to the electrical resistor, capacitor and inductor in the other physical domains. Resistive analogs dissipate energy:

$$w_R(t) \;=\; \int_{-\infty}^{t} p_R(t')\,dt' \;=\; \int_{-\infty}^{t} \underset{\text{through}}{i(t')}\;\underset{\text{across}}{R\,i(t')}\,dt' \;=\; R\int_{-\infty}^{t} i(t')^2\,dt' \quad \text{[J].}$$

Capacitive analogs store energy of across type:

$$w_C(t) \;=\; \int_{-\infty}^{t} p_C(t')\,dt' \;=\; \int_{-\infty}^{t} \underset{\text{through}}{C\frac{dv}{dt'}(t')}\;\underset{\text{across}}{v(t')}\,dt' \;=\; C\int_{0}^{v(t)} v\,dv \;=\; \frac{1}{2}C\underset{\text{across}}{v(t)^2} \quad \text{[J].}$$

Inductive analogs store energy of through type:

$$w_L(t) \;=\; \int_{-\infty}^{t} p_L(t')\,dt' \;=\; \int_{-\infty}^{t} \underset{\text{through}}{i(t')}\;\underset{\text{across}}{L\frac{di}{dt'}(t')}\,dt' \;=\; L\int_{0}^{i(t)} i\,di \;=\; \frac{1}{2}L\,\underset{\text{through}}{i(t)^2} \quad \text{[J].}$$

The mechanical-translational lumped components are the mechanical damper, the mechanical mass and the mechanical compliance (spring):

Mechanical Damper: The law for viscous damping is

$$u \;=\; \frac{1}{B}f \qquad \text{[m/s]}$$

where $\frac{1}{B}$, analogous to electrical resistance, is called the mechanical responsiveness r_M [m/(s.N)]. ($B \;=\; \frac{f}{u}$ is called the mechanical resistance R_M.)

Mechanical Mass: Note that the velocity u is always reckoned with respect to the system inertial frame (e.g. the earth). Hence the mass component cannot be "floated". Newton's law for mass is

$$f(t) \;=\; M\frac{du}{dt}(t) \quad \text{or} \quad u(t) \;=\; \frac{1}{M}\int_{-\infty}^{t} f(t')\,dt'$$

where M, analogous to electrical capacitance, is the mechanical mass M_M [kg].

Mechanical Compliance: Hooke's law for compliance is

$$u(t) \;=\; \frac{1}{K}\frac{df}{dt}(t) \quad \text{or} \quad f(t) \;=\; K\int_{-\infty}^{t} u(t')\,dt' \;=\; K\,x(t)$$

where $\frac{1}{K}$, analogous to electrical inductance, is called the mechanical compliance C_M [m/N]. ($K \;=\; \frac{f}{x}$ is called the mechanical stiffness, x [m] being the translational displacement.)

The acoustical lumped components are the acoustical damper, the acoustical mass and the acoustical compliance:

Acoustical Damper: The law for viscous damping is

$$p = R_A U \quad \text{[Pa]}$$

The acoustical resistance R_A [N.s/m^5] is analogous to electrical resistance.

Acoustical Compliance: Note that the pressure difference p is always reckoned with respect to the quiescent gas pressure (e.g. the atmospheric pressure). Hence the acoustic compliance component cannot be "floated". Adiabatic compression of a gas-filled chamber or box of volume V_B (ignoring acceleration of the gas particles) follows the law

$$U(t) = C_A \frac{dp}{dt} \quad \text{or} \quad p(t) = \frac{1}{C_A}\int_{-\infty}^{t} U(t')\,dt'$$

The acoustical compliance C_A [m^5/N] is analogous to electrical capacitance. Also, for air,

$$C_A = \frac{V_B}{\rho_0 c^2} = \frac{V_B}{\gamma P_0} = \frac{V_B}{141855} \quad \text{[m}^5\text{/N]}$$

under standard conditions whereby the atmospheric pressure $P_0 = 101325$ Pa and the temperature is 20°C (293.15 K). Then the air density $\rho_0 = 1.204084$ kg/m^3 and the speed of sound $c = 343.237$ m/s. Since air is a mixture of mainly diatomic gases, the ratio of the specific heat of air at constant pressure to the specific heat at constant volume is $\gamma = 1.400$.

Acoustical Mass: Acceleration of gas through a tube (ignoring compression) follows Newton's law

$$p(t) = M_A \frac{dU}{dt}(t) \quad \text{or} \quad U(t) = \frac{1}{M_A}\int_{-\infty}^{t} p(t')\,dt'$$

The acoustical mass M_A [kg/m^4] is analogous to electrical inductance. Also, for air,

$$M_A = \frac{\rho_0 \ell_T}{S_T} \quad \text{[kg/m}^4\text{]}$$

where ℓ_T is the effective length of the tube and S_T is its cross-sectional area.

For an ideal transformer of turns ratio n

$$v_1 = n v_2$$
$$i_1 = \frac{1}{n}(-i_2)$$

where both currents are referenced as flowing into the positive (dotted) terminals.

For an ideal gyrator of gyro-resistance (or gyro-ratio) r [Ω]

$$v_1 = r(-i_2)$$
$$i_1 = \frac{1}{r} v_2$$

where again both currents are referenced as flowing into the positive terminals. In contrast to a transformer, a gyrator interchanges the through and the across variables between its primary and secondary sides. The ideal gyrator is a non-reciprocal device since interchanging sides (1↔2) means ($r \leftrightarrow -r$). Thus the gyrator symbol includes an arrow directed from the primary side to the secondary side.

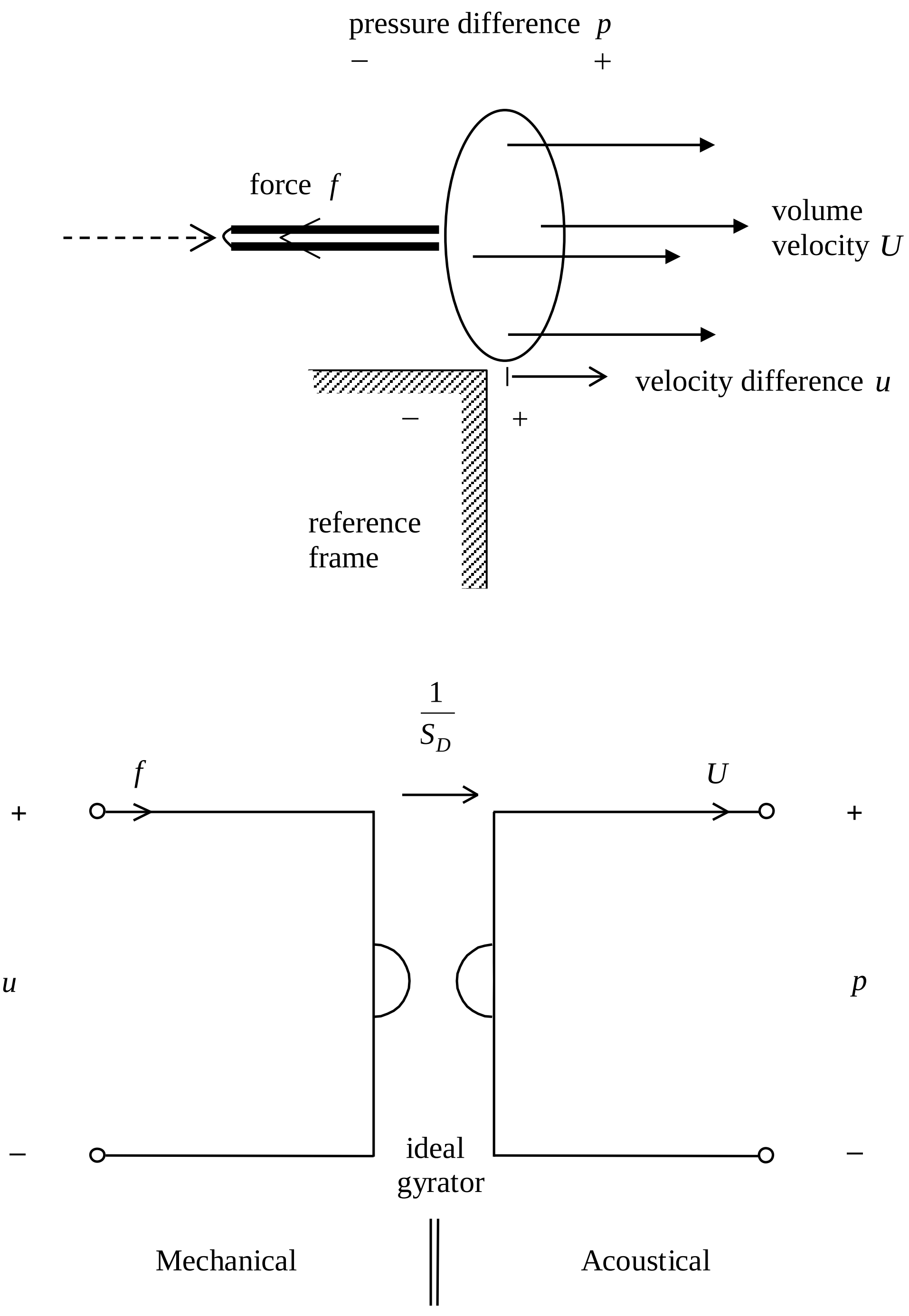


Figure A1: An ideal piston in an infinite baffle acting as a mechano-acoustic transducer.

A good example of a gyrator is an ideal piston or diaphragm in an infinite baffle which acts as a mechano-acoustic transducer. The ideal piston, but not the baffle, is shown in Figure A1. The corresponding equations are

$$u = \frac{1}{S_D} U$$
$$f = S_D\, p$$

where S_D [m^2] is the area of the diaphragm and where the volume velocity U is referenced as an outflow (rather than an inflow) on the secondary side of the gyrator. Note also that the arrow depicting the mechanical through variable of force is actually the piston's reaction force (rather than the applied compressive force) in order to match the associated reference direction of the mechanical across variable (the velocity difference). The gyro-ratio is therefore $r = \frac{1}{S_D}$ [m^{-2}] for the mechano-acoustic transducer.

The equivalent circuit of an electrostatic loudspeaker also involves a gyrator between the electrical domain and the mechanical domain. The cross-section of a push-pull electrostatic panel is shown in Figure A2(a). The diaphragm is typically charged via a large series resistance R_0 from a high-voltage source V_0 which is referenced to the centre-tap of the audio signal voltage source v that is applied between the perforated stators.

Let S_D be the area of the diaphragm and each stator and let Q be the charge on the membrane. With the diaphragm in its central reference position, the separation between the diaphragm and either stator is d and the induced charge on either stator is $-Q/2$. When an audio signal voltage $v(t)$ is applied, an audio signal charge $q(t) = \int_{-\infty}^{t} i(t')\,dt'$ is added to the left-hand stator and subtracted from the right-hand stator where $i(t)$ is the audio signal current. Furthermore, the diaphragm will move by some distance $x(t) = \int_{-\infty}^{t} u(t')\,dt'$ to the right from its central reference position where $u(t)$ is diaphragm velocity in response to the electrostatic force $f(t)$.

Using the formula for the capacitance of a parallel-plate capacitor, the "static" capacitance between the stators is

$$C_0 = \frac{\varepsilon_0 S_D}{2d} \quad \text{[F]}.$$

Here $\varepsilon_0 \approx 8.854\times 10^{-12}$ [F/m] is the permittivity of free space. The capacitance between the diaphragm and either stator is therefore $2C_0$ and so the charge on the diaphragm is

$$Q = 4 C_0 V_0 \quad \text{[C]}.$$

From Gauss' flux law the electric field intensity between the left-hand stator and the diaphragm is

$$E_\ell = \frac{q - Q/2}{2\varepsilon_0 S_D} \quad \text{[V/m]}$$

while the electric field intensity between the diaphragm and the right-hand stator is

$$E_r = \frac{q + Q/2}{2\varepsilon_0 S_D} \quad \text{[V/m]}$$

both referenced to the right. By Coulomb's law the net electrostatic force on the diaphragm (referenced to the right) is therefore

$$f \;=\; \left(E_\ell + E_r\right)Q \;=\; \frac{Q}{\varepsilon_0 S_D}q \quad \text{[N]}.$$

Also the audio signal voltage between the stators must equal

$$v \;=\; E_\ell(d+x) + E_r(d-x) \;=\; \frac{1}{\varepsilon_0\, S_D}(2d\,q - Q\,x) \quad \text{[V]}$$

so that

$$q \;=\; \frac{\varepsilon_0 S_D}{2d}v \;+\; \frac{Q}{2d}x \quad \text{[C]}$$

and

$$x \;=\; \frac{1}{Q}(2d\,q \;-\; \varepsilon_0 S_D\, v) \quad \text{[m]}.$$

Therefore

$$\begin{aligned} f(t) \;&=\; \frac{Q}{\varepsilon_0 S_D}\left(\frac{\varepsilon_0 S_D}{2d}v(t) \;+\; \frac{Q}{2d}x(t)\right) \;=\; \frac{Q}{2d}v(t) \;+\; \frac{Q^2}{2d\varepsilon_0 S_D}x(t) \\ &=\; \frac{Q}{2d}v(t) \;+\; \frac{1}{C_0\left(\frac{2d}{Q}\right)^2}\int_{-\infty}^{t} u(t')\,dt' \qquad \text{[N]} \end{aligned}$$

and

$$\begin{aligned} u(t) \;&=\; \frac{dx}{dt}(t) \;=\; \frac{1}{Q}\left(2d\,\frac{dq}{dt}(t) \;-\; \varepsilon_0 S_D\,\frac{dv}{dt}(t)\right) \\ &=\; \frac{2d}{Q}\left(i(t) \;-\; C_0\,\frac{dv}{dt}(t)\right) \end{aligned} \qquad \text{[m/s]}.$$

These equations describe the equivalent circuit in Figure A2(b). The gyro-ratio of the ideal gyrator is $r = \dfrac{2d}{Q}$ [m/C]. The capacitor on the electrical side of the gyrator represents the "static" capacitance C_0 [F] between the stators. The inductor on the mechanical side of the gyrator represents a negative mechanical compliance $C_{ME} = -C_0\left(\dfrac{2d}{Q}\right)^2$ [m/N] caused by the attraction of the charged diaphragm to the nearer stator whenever it excurses away from its central reference position. To achieve diaphragm stability the negative mechanical compliance C_{ME} must be counteracted by a smaller positive mechanical compliance C_{MS} from the diaphragm suspension, which, for an elastic diaphragm, is usually set by prestretching the diaphragm. The net mechanical compliance would then be

$$C_{MT} \;=\; \frac{C_{ME}\,C_{MS}}{C_{ME} + C_{MS}} \;=\; \frac{C_{MS}}{1-k^2} \quad \text{[m/N]}$$

where k^2, called the coefficient of electromechanical coupling [3, page 198], must be less than one:

$$k^2 = -\frac{C_{MS}}{C_{ME}} = \frac{C_{MS}}{C_0}\left(\frac{Q}{2d}\right)^2 < 1.$$

To complete the equivalent circuit of an electrostatic panel, the mechanical mass M_{MD} [kg] of the diaphragm would be included as a parallel capacitor on the mechanical side of the gyrator and finally the acoustical loading would be added by attaching the equivalent circuit of the diaphragm acting as a mechano-acoustic transducer and incorporating the radiation impedances for each side of the diaphragm.

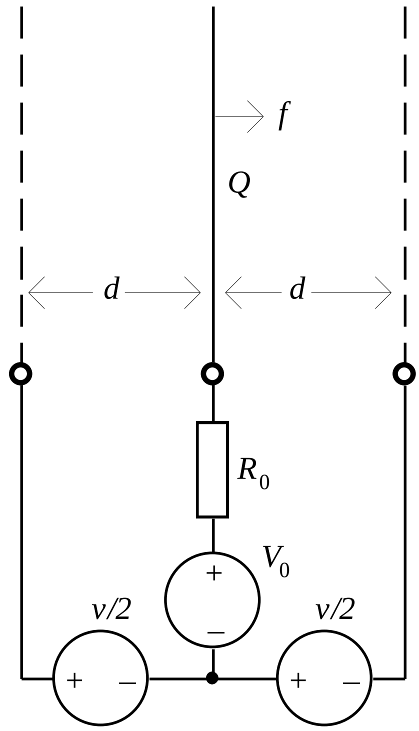


Figure A2(a): Cross-section of a push-pull electrostatic-mechanical transducer.

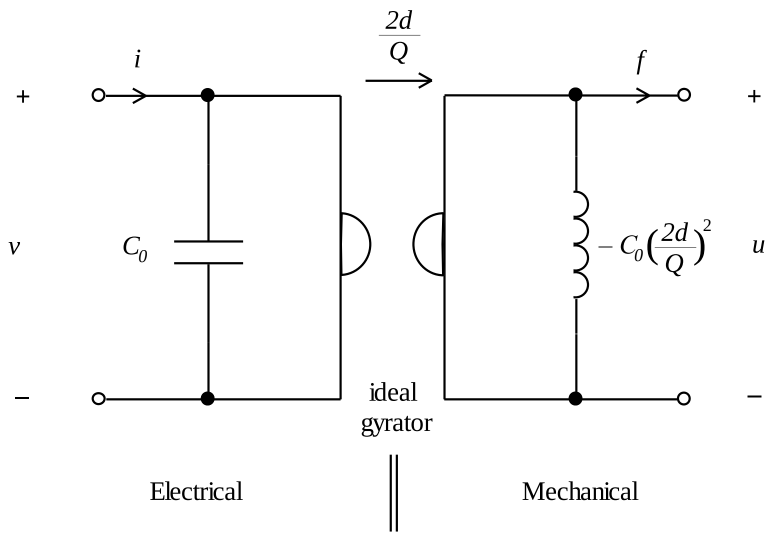
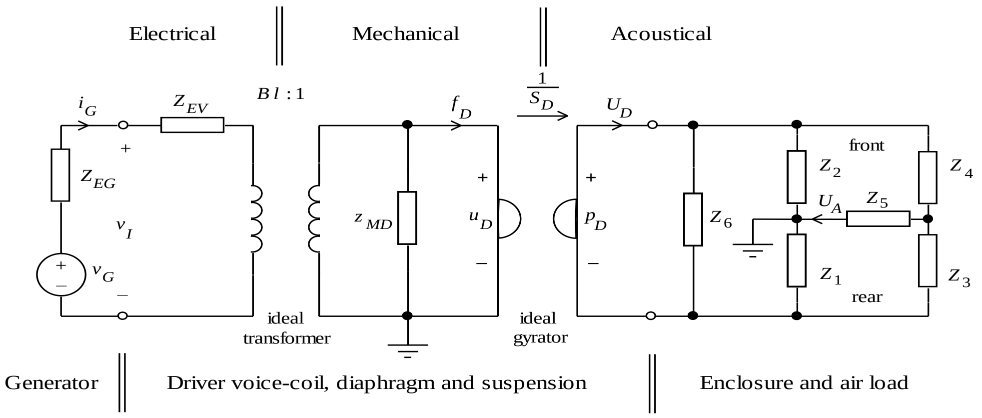


Figure A2(b): Equivalent circuit of a push-pull electrostatic-mechanical transducer.



$$Z_{EV} = R_{EV} + j\omega L_{EV}$$

$$\frac{1}{z_{MD}} = \frac{1}{j\omega C_{MS}} + \frac{1}{r_{MS}} + j\omega M_{MD}$$

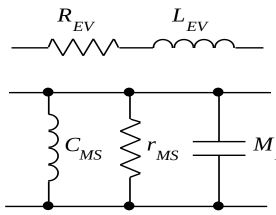
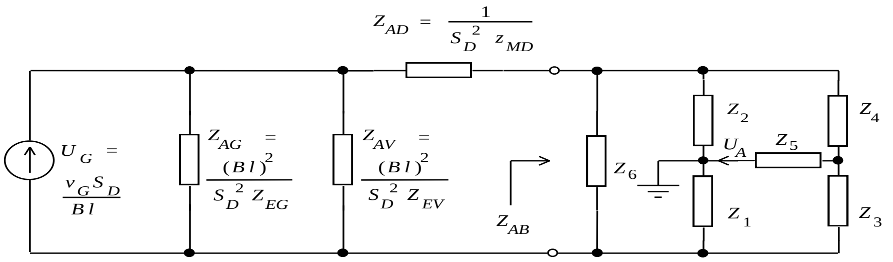


Figure A3(a): Mixed equivalent circuit of magnetostatic loudspeaker system.



$$Z_{AD} = \frac{1}{S_D^2 \, z_{MD}} = \frac{1}{j\omega C_{AS}} + R_{AS} + j\omega M_{AD}$$

where $C_{AS} = S_D^2 C_{MS}$, $R_{AS} = \dfrac{1}{S_D^2 \, r_{MS}}$, $M_{AD} = \dfrac{M_{MD}}{S_D^2}$

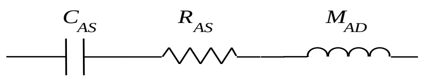


Figure A3(b): Acoustical equivalent circuit of magnetostatic loudspeaker system.

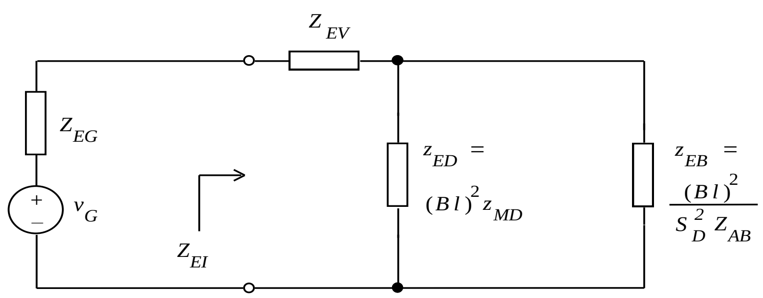


Figure A3(c): Electrical equivalent circuit of magnetostatic loudspeaker system.

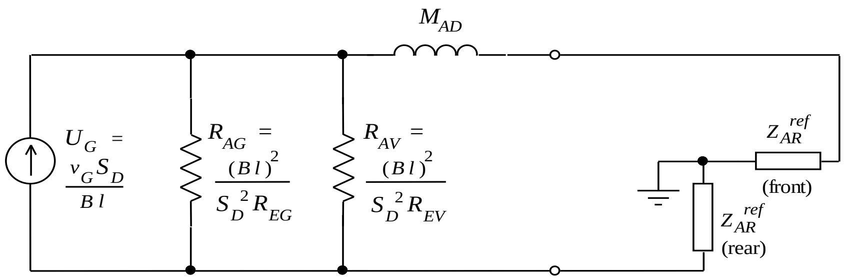


Figure A3(d): Acoustical equivalent circuit for calculation of reference SPL of magnetostatic loudspeaker system.

Figure A3(a) shows the development of the equivalent circuit of the magnetostatic loudspeaker. It involves an ideal transformer between the electrical domain and the mechanical domain. The turns ratio of the ideal transformer is the force factor $B\ell$ [Tm].

The mixed equivalent circuit can be transformed into just one domain as shown in Figures A3(b) and A3(c).

The midband reference SPL of the magnetostatic loudspeaker is found from the simplified equivalent circuit shown in Figure A3(d).

Appendix B: Relevant Literature

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